A Wavelet Based Time Domain Moment Method for the Analysis of Three-dimensional Electromagnetic Fields

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ABSTRACT

A novel time domain simulation approach for analyzing real three-dimensional (3-D) passive microwave structures is presented. The electromagnetic fields of the structure under investigation are expanded into a hierarchical system of wavelets and scaling functions for all three dimensions leading to a stable, multiscale algorithm. By neglecting small wavelet coefficients and thereby reducing the computational effort, this method is equivalent to a finite difference in time domain scheme with a time dynamic space adaptive grid. An overview of the main ideas and advantages of this new method as well as the results of two numerical examples are given.

I. INTRODUCTION

In recent years time domain techniques for solving Maxwell's equations in form of an initial boundary value problem have become very popular. The most common techniques are the finite-difference time-domain (FDTD) [l] and the transmission-line matrix (TLM) [2] methods. Both methods use samples in time and space of field quantities at predefined grid points to approximate the physical continuum. The chosen grid is of crucial importance for the accuracy of the solution and the numerical expense of the simulation. The discretization of the structure must be sufficient for all field distributions that occur during the simulation process. The approach presented here enables a time dynamic spatial discretization of Maxwell's differential equations that automatically adapts to the local regularity of the solution.

II. THEORY

Starting with Maxwell's curl equations in the time domain for non-conducting isotropic media

$$\frac{\partial}{\partial t}\mathbf{E} = \frac{1}{\varepsilon}\mathrm{curl}\mathbf{H}, \quad \frac{\partial}{\partial t}\mathbf{H} = -\frac{1}{\mu}\mathrm{curl}\mathbf{E}, \qquad (1)$$

then the time derivatives are approximated with central finite differences of second order so explicit equations with respect to time are obtained:

$$\mathbf{E}^{n} = \mathbf{E}^{n-1} + \frac{\Delta t}{\varepsilon}\mathrm{curl}\mathbf{H}^{n-\frac{1}{2}},$$

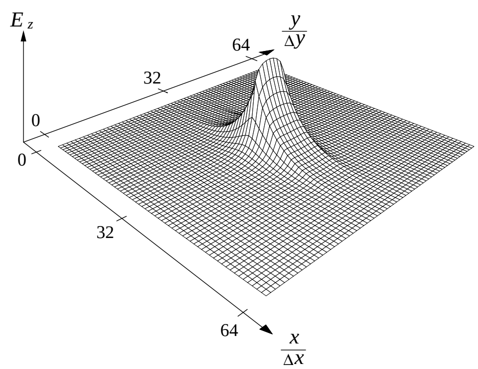
$$\mathbf{H}^{n+1/2} = \mathbf{H}^{n-1/2} + \frac{\Delta t}{\mu}\,\mathrm{curl}\mathbf{E}^{n}. \qquad (2)$$

The field $\mathbf{E}^{n}$ is the electric field strength at time step *n*, $\mathbf{H}^{n+\frac{1}{2}}$ the magnetic field strength at time step *n*+1/2. Then the electromagnetic field in time domain is expanded into a system of orthonormal basis functions. These basis functions are derived from all permutations that can be constructed as a product of three one-dimensional (1-D) compactly supported Daubechies wavelets $\psi_k^j(\cdot)$ or scaling functions $\varphi_k^j(\cdot)$ [4]. All these permutations form a complete orthonormal basis for $L^2(\mathbf{R}^3)$.

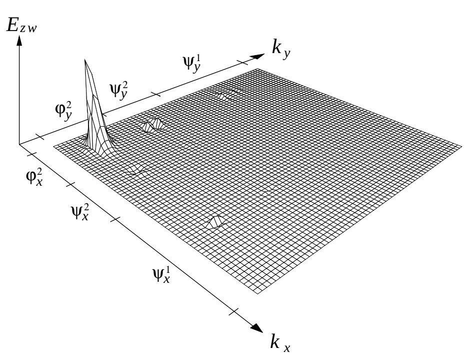
The fast wavelet transform and all techniques used in the procedure are leading to fast and simple algorithms. In contrast to the multiresolution time domain (MRTD) scheme presented in [5], [6], no integrals have to be evaluated.

The multiresolution representation of the field solution can be used to reduce the number of unknowns. For areas with a smooth field distribution, wavelet coefficients that describe finer details become very small and can therefore be neglected. This is demonstrated in figure 1a) to 1c) where the field distribution (a), the wavelet coefficients (b) and the considered wavelet coefficients (c) of an excitation point on a microstrip line are shown.

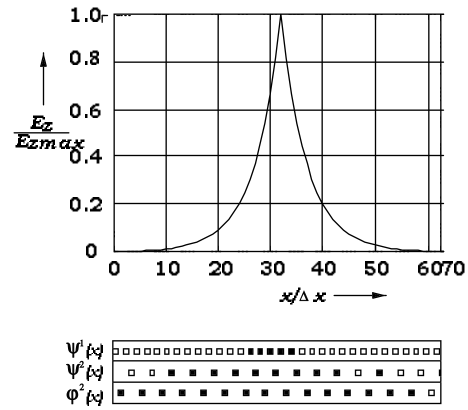
This is equivalent to a local reduction of the discretization density. In the vicinity of field singularities or sharp transitions the corresponding wavelet coefficients with small dilation parameters must be taken into account because they contribute significantly to the solution, so the full discretization density is maintained in these regions. During the simulation process a sequence of field solutions is calculated. For every time step n only the wavelet coefficients that are larger than a given threshold λ_q in time step q and all adjacent coefficients are considered. The number of discrete time steps $\Delta q = n - q$ must be equal or smaller than $\Delta q_{\max} = \Delta s / (v_{ph} \Delta t)$ in order to track propagating waves. Δs denotes the finest discretization density and v_{ph} is the phase velocity. This means that it is sufficient to test negligible wavelet coefficients every $q_{\max}$ time steps and to use this information for the subsequent time steps. Besides,



a)



b)



c)

Fig. 1: a) Field distribution on a microstrip line.
b) The wavelet coefficients of the field shown near the excitation point in Fig. 1a.
c) Considered (▮▮) and neglected ( ) wavelet coefficients.

the threshold is recalculated every $q_{\max}$ time steps as a fraction of the energy of all wavelet coefficients.

To utilize the compressed multiscale representation of the electromagnetic field, (2) must be expressed in terms of wavelets and scaling functions. For this purpose two operators are introduced. The first one is the curl operator that can simply be divided into consecutive applications of derivative operators. The second operator accounts for the material distribution. Using these operators (2) can be rewritten:

$$\mathbf{E}_w^n = \mathbf{E}_w^{n-1} + \Delta t M_\varepsilon(\varepsilon) C\left(\mathbf{H}_w^{n-1/2}\right),$$
$$\mathbf{H}_w^{n+1/2} = \mathbf{H}_w^{n-1/2} + \Delta t M_\mu(\mu) C\left(\mathbf{E}_w^n\right), \tag{3}$$

where $\mathbf{E}_w$ and $\mathbf{H}_w$ represent the wavelet expansion of the electromagnetic fields. M and C denote the material and curl operator, respectively. To obtain a stable algorithm, the time step Δt must be determined according to the derivative operator [7]. For second-order operators the maximum stable time step can be obtained from the stability condition of the FDTD method. For higher-order derivative operators the stability condition becomes more restrictive, but the numerical dispersion decreases [9], [10] so that the discretization density can be reduced. Perfect electric or magnetic walls are modeled utilizing the symmetry conditions for the tangential fields, which are taken into account for the construction of the derivative operators. In addition, absorbing boundary algorithms can be realized as operators. In a first approach the absorbing boundary condition of first-order proposed by Mur [11] has been implemented into the developed software.

The algorithm that results was found to be stable for all numerical tests and applications. In addition, the method does not suffer from the excitation of spurious modes like the MRTD method [5], [6] which up to now apparantly only has been applied to two-dimensional problems [6]. A further advantage in comparison to the MRTD method is the use of compactly supported Daubechies wavelets..

III. NUMERICAL RESULTS

To verify the new method, two typical three-dimensional microwave structures have been investigated. The first example is a dielectric post discontinuity in a WR-90 waveguide. The geometry of the structure as well as a comparison of measured and simulated results are shown in Fig. 2.

The discontinuity was simulated by applying a derivative operator of second order. As basis functions compactly supported Daubechies wavelets and scaling functions with two vanishing moments were used. Three different scales were taken into account, so the ratio of the finest and coarsest discretization density in one direction is equal to four. Assuming the finest discretization density, the width of the waveguide was discretized using 58 wavelet coefficients. The length of the dielectric post was discretized with 16 coefficients. The two waveguide ports were terminated with the absorbing boundary condition proposed by Mur (first order) [11] combined with super absorption [12]. The systematic error caused by the imperfect boundary condition was corrected using the procedure presented in [13].

The second structure under investigation is a meander line structure on alumina substrate which was first investigated by Wertgen [14]. The layout of the microstrip discontinuity and the return loss are depicted in Fig. 3.

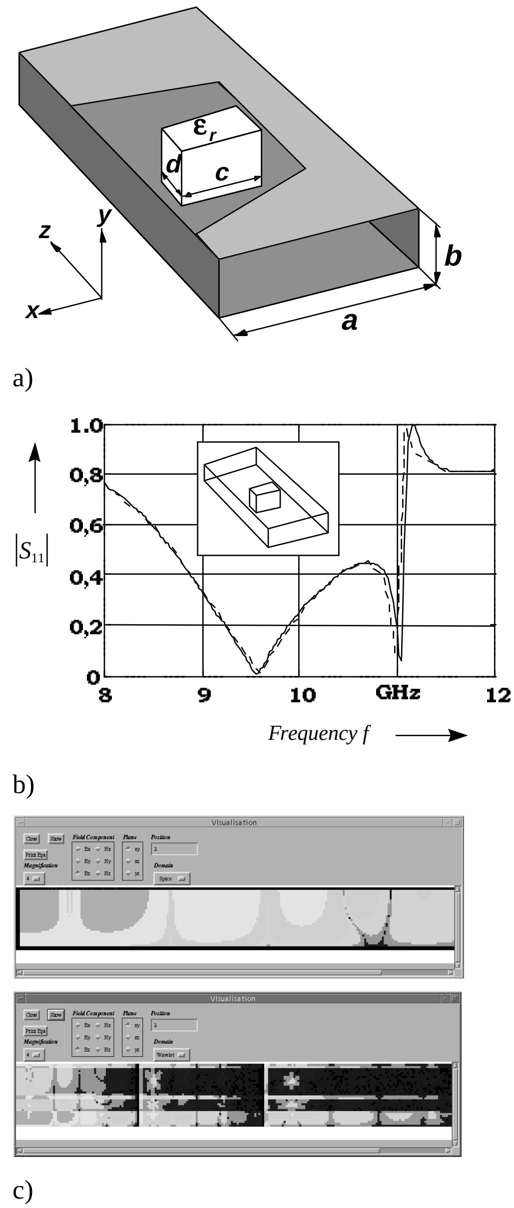
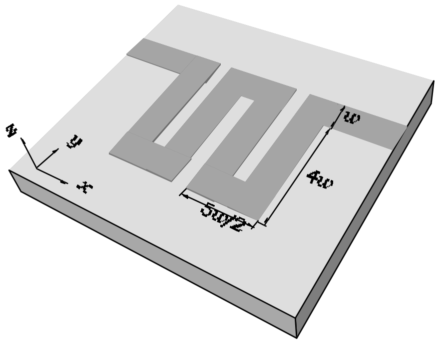
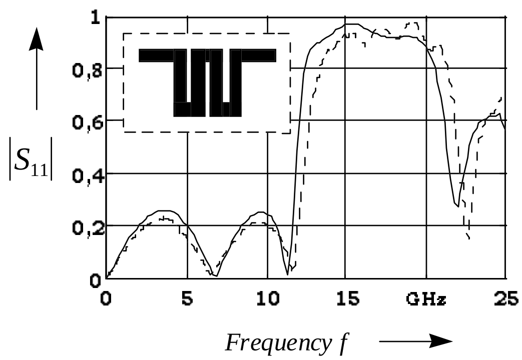


Fig. 2: a) A 3D-waveguide problem with dielectric insert. B) Comparison of simulated (——) and measured (- - - -) results for the reflexion coefficient of the structure shown in Fig. 2a). c) Distribution of the wave-let coefficients of the structure.

The same simulation parameters as for the first example were used. In addition the same type of absorbing boundary condition was applied. The finest discretization for the width of the microstrip line and the substrate thickness were four and six



a)



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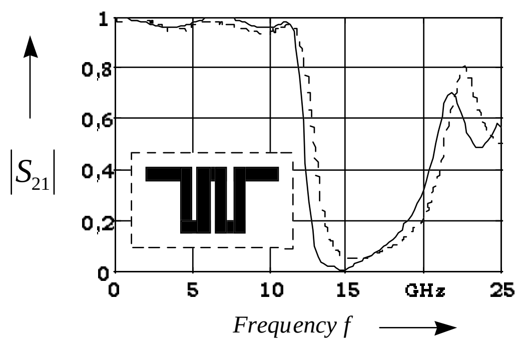

c)

Fig. 3: a) A microstrip meander line test structure [14]. b) Comparison of simulated (—) and measured (- - -) reflexion coefficient of the meander line. c) Comparison of simulated (—) and measured (- - -) transmission coefficient of the meander line.

wavelet coefficients, respectively. Except for a small frequency shift in the case of the meander line (which is known also to occur in comparison to other numerical techniques, so that probably the measurement is erroneous), the comparison of the simulated and measured results show excellent agreement for both examples.

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